Formal contradiction arising from a universal form of Godel's Theorem

One-step Gödel theorem

Let *K* be *any* a consistent, sufficiently strong logic. Then

$G_0 \qquad (\exists X)(\Sigma \nvdash_K X \text{ and } \Sigma \vDash_K X)$

In words, "There exists at a statement *X*, such that there is no proof of *X* in *K* from the axioms Σ, but *X* is a consequence in *K* of Σ." There may be many such statements *X*, but Godel's theorem explicitly constructs one such statement, $Q = \Sigma \nvdash_K Q$. Q is an instance of G_0. Allowing the universalization of this statement, we obtain:

Universal Gödel theorem

Let *K* be *any* a consistent, sufficiently strong logic. Then

$G \qquad (\forall \Sigma)(\exists X)(\Sigma \nvdash_K X \text{ and } \Sigma \vDash_K X)$

In words, "Given a sufficiently strong logic *K*, then for **all** extensions of *K* formed by adjoining new axioms to *K* to form a set of axioms Σ there exists at a statement *X*, such that there is no proof of *X* in *K* from the axioms Σ, but *X* is a consequence in *K* of Σ."

Formal contradiction

$\Sigma * \vdash_K G$	
$\Sigma * \vdash_K (\forall \Sigma)(\exists X)(\Sigma \nvdash_K X \text{ and } \Sigma \vDash_K X)$	Substituting for G
$\Sigma * \vdash_K (\exists X)(\Sigma * \nvdash_K X \text{ and } \Sigma * \vDash_K X)$	Universal instantiation, $\Sigma = \Sigma *$
$\Sigma * \vdash_K (\Sigma * \nvdash_K Q \text{ and } \Sigma * \vDash_K Q)$	Where Q is the specific Godel proposition
$\Sigma * \vdash_K \Sigma * \nvdash_K Q \text{ and } \Sigma * \vdash_K \Sigma * \vDash_K Q$	
$\Sigma * \vdash_K \nvdash_K Q \text{ and } \Sigma * \vdash_K \vDash_K Q$	
$\Sigma * \nvdash_K Q \text{ and } \Sigma * \vdash_K Q$	

The assumption that the **universal** Gödel's theorem is one of the first-order logics leads to a formal contradiction, albeit that the assumption that the **one-step** Gödel's theorem is first-order does not entail such a contradiction.